

# Intrinsic uncertainty on the nature of dark energy

Wessel Valkenburg,<sup>1,2</sup> Martin Kunz,<sup>3,4</sup> and Valerio Marra<sup>2</sup>

<sup>1</sup>*Instituut-Lorentz for Theoretical Physics, Universiteit Leiden,  
Postbus 9506, 2333 CA Leiden, The Netherlands*

<sup>2</sup>*Institut für Theoretische Physik, Universität Heidelberg, Philosophenweg 16, 69120 Heidelberg, Germany*

<sup>3</sup>*Département de Physique Théorique and Center for Astroparticle Physics,  
Université de Genève, Quai E. Ansermet 24, CH-1211 Genève 4, Switzerland*

<sup>4</sup>*African Institute for Mathematical Sciences, 6-8 Melrose Road, Muizenberg, Cape Town, South Africa*

We argue that there is an intrinsic noise on measurements of the equation of state parameter  $w = p/\rho$  from large-scale structure around us. The presence of the large scale structure leads to an ambiguity in the definition of the background universe and thus there is a maximal precision with which we can determine the equation of state of dark energy. To study the uncertainty due to local structure, we model density perturbations stemming from a standard inflationary power spectrum by means of the exact Lemaître-Tolman-Bondi solution of Einstein's equation, and show that the usual distribution of matter inhomogeneities in a  $\Lambda$ CDM cosmology causes a variation of  $w$  – as inferred from distance measures – of several percent.

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**Introduction** A key quantity to characterize the nature of the dark energy is its equation of state parameter  $w = p/\rho$ . Current and future cosmological observations try to measure  $w$  ever more accurately, and the power of dark-energy missions is judged by the minimal error that they can achieve on the dark-energy equation of state parameter  $w = p/\rho$ . This is for example the basis of the Dark Energy Task Force (DETF) [1] Figure of Merit (FoM), which is given by the determinant of the Fisher matrix for the parameters  $w_0$  and  $w_a$  in a linear parameterization of the equation state,  $w(a) = w_0 + (1 - a)w_a$ . An important question in the context of dark-energy research is then whether there is a natural limit for the precision with which  $w$  can be measured, or whether one can in principle determine  $w(a)$  to an arbitrary precision.

In this letter we argue that the matter fluctuations that are always present in the universe provide such a limit, and we determine the unavoidable variation of  $w(a)$  as expected in the  $\Lambda$ CDM concordance model (for which in principle  $w = -1$ ). Those variations appear because we always measure *any* observable quantity in the true perturbed universe, even if we consider “background” quantities like the luminosity distance (see e.g. [2] and references therein), and they remain significant even when averaging over angles [3]. This is a direct manifestation of the “fitting problem” [4], i.e. the attempt to fit a homogeneous and isotropic FLRW model to a lumpy universe [5–7] rather than directly modeling the inhomogeneities [8]. If, on one hand, fluctuations in e.g. the luminosity distance allow us in principle to obtain additional cosmological information (see e.g. [9–13]), on the other hand they result in an intrinsic noise in the determination of cosmological parameters. Indeed, although the perturbations in the metric are small, only about  $10^{-5}$ , they can be amplified when going to quantities that involve derivatives like  $w(a)$ , as demonstrated in e.g. [14].

The outline of this letter is as follows: first we define the probability of inhomogeneities. Then we explain how we model said inhomogeneities. Next we show the noise caused by this structure on the evolution of the dark-energy equation of state inferred by an observer who ignores the inhomogeneity. Finally we obtain the error in the cosmological parameters measured by an observer who fits the luminosity distance under various assumptions.

**Probability of a local structure** The root mean square of the density perturbation in a sphere of radius  $L$  around any point today in a Gaussian density field is [15]

$$\sigma_L = \left[ \int_0^\infty dk \frac{k^2}{2\pi^2} P(k) \left( \frac{3j_1(Lk)}{Lk} \right)^2 \right]^{\frac{1}{2}}, \quad (1)$$

where  $P(k)$  is the matter power spectrum today as a function of wavenumber  $k$ , and  $j_1$  is the spherical Bessel function of the first kind.

Given some inhomogeneity with a mass  $M(L)$  inside a radius  $L$ , one can define the average density perturbation  $\delta_0 \equiv M(L)/\bar{M}(L) - 1$  relative to the homogeneous background which predicts a mass  $\bar{M}(L)$  inside the same radius  $L$ . Then the probability of having such a structure is [16],

$$P(\delta_0|L) = \frac{1}{\sigma_L \sqrt{2\pi}} e^{-\frac{\delta_0^2}{2\sigma_L^2}}. \quad (2)$$

If for simplicity we model the inhomogeneity spherically symmetric, that is, we presume that the central observer averages observations over directions, the mass is given by  $M(r) \equiv 4\pi \int_0^r dr \sqrt{-g} \rho_m(r)$  with  $g$  the determinant of the metric.

**Model for local inhomogeneity** To model the inhomogeneity averaged over angular directions, we adopt the spherically symmetric Lemaître-Tolman-Bondi solution [17–19] including a cosmological constant  $\Lambda$  (ALTB, see e.g. [20–23]), for which we can compute all distance measures exactly.

The ALTb metric in the comoving and synchronous gauge can be written as (using units for which  $c = 1$ )

$$ds^2 = -dt^2 + \frac{a_{\parallel}^2(t, r)}{1 - k(r)r^2} dr^2 + a_{\perp}^2(t, r)r^2 d\Omega^2, \quad (3)$$

where the longitudinal ( $a_{\parallel}$ ) and perpendicular ( $a_{\perp}$ ) scale factors are related by  $a_{\parallel} = (a_{\perp}r)'$ , and a prime denotes partial derivation with respect to the coordinate radius  $r$ . In the limit  $k \rightarrow \text{const.}$ , and  $a_{\perp} = a_{\parallel}$  we recover the FLRW metric, but in a LTB metric the curvature  $k(r)$  is a free function and in general is not constant.

The two scale factors define two different Hubble rates:

$$H_{\perp}(t, r) \equiv \frac{\dot{a}_{\perp}}{a_{\perp}}, \quad H_{\parallel}(t, r) \equiv \frac{\dot{a}_{\parallel}}{a_{\parallel}}. \quad (4)$$

The analogue of the Friedmann equation in this space-time can be written in a familiar form,

$$\frac{H_{\perp}^2}{H_{\perp 0}^2} = \Omega_m a_{\perp}^{-3} + \Omega_k a_{\perp}^{-2} + \Omega_{\Lambda}, \quad (5)$$

where we adopted the gauge fixing  $a_{\perp 0} = 1$ . However, the density parameters are now also functions of  $r$ ,

$$\Omega_m(r) = \frac{m(r)}{H_{\perp 0}^2}, \quad \Omega_k(r) = -\frac{k}{H_{\perp 0}^2}, \quad \Omega_{\Lambda}(r) = \frac{\Lambda}{3H_{\perp 0}^2}, \quad (6)$$

so as to satisfy  $\Omega_m(r) + \Omega_k(r) + \Omega_{\Lambda}(r) = 1$ . The free function  $m(r)$  is related to the local matter density  $8\pi G \rho_m(t, r) = (mr^3)' / a_{\parallel} a_{\perp}^2 r^2$ . The matter density obeys the usual conservation equation  $\dot{\rho}_m + (2H_{\perp} + H_{\parallel})\rho_m = 0$ .

Finally, time  $t$  and radius  $r$  as a function of redshift  $z$  are determined on the past light cone of the central observer by the differential equations for radial null geodesics,

$$\frac{dt}{dz} = -\frac{1}{(1+z)H_{\parallel}}, \quad \frac{dr}{dz} = \frac{\sqrt{1 - kr^2}}{(1+z)a_{\parallel}H_{\parallel}}, \quad (7)$$

with the initial conditions  $t(0) = t_0$  and  $r(0) = 0$ . The area ( $d_A$ ) and luminosity ( $d_L$ ) distances are given by

$$d_A(z) = a_{\perp}(t(z), r(z)) r(z), \quad d_L = (1+z)^2 d_A. \quad (8)$$

**Density profile** The age of the universe is a function of  $(t, r)$  and is obtained by integrating the Friedmann equation (5) from the big-bang time  $t_{\text{bb}}(r)$  to time  $t$ :

$$t - t_{\text{bb}} = \frac{1}{H_{\perp 0}(r)} \int_0^{a_{\perp}(t, r)} \frac{dx}{\sqrt{\Omega_m(r)x^{-1} + \Omega_k(r) + \Omega_{\Lambda}(r)x^2}}. \quad (9)$$

Eq. (9) relates the three free functions  $t_{\text{bb}}$ ,  $k(r)$  and  $m(r)$ , so that density of the dust field in the ALTb model is specified by two free functional degrees of freedom, where we choose  $k(r)$  and  $t_{\text{bb}}(r)$ . Any radial dependence of  $t_{\text{bb}}(r)$  is directly related to a decaying mode in the matter density field [24, 25]. By choosing  $t_{\text{bb}}(r) = 0$  decaying modes are absent, in agreement with the standard scenario of inflation.

We parameterize the curvature function with the monotonic profile

$$k(r) = k_b + (k_c - k_b) P_3(r/r_b), \quad (10)$$

where  $r_b$  is the comoving radius of the spherical inhomogeneity and  $P_3$  is the function

$$P_n(x) = \begin{cases} 1 - e^{-(1-x)^n/x} & \text{for } 0 \leq x < 1 \\ 0 & \text{for } x \geq 1 \end{cases}$$

for  $n = 3$ . The function  $P_n(x)$  interpolates from 1 to 0 when  $x$  varies from 0 to 1 while remaining  $n$  times differentiable, which implies that  $k(r)$  is  $C^n$  everywhere. We choose  $n = 3$ , such that the metric is  $C^2$  and the Riemann curvature is  $C^0$ . For  $r \geq r_b$  the curvature profile equals the curvature  $k_b$  of the background such that there the metric reduces exactly to the  $\Lambda$ CDM model. The central under- or over-density, determined by the curvature  $k_c$  at the center, is automatically compensated by a surrounding over- or under-dense shell. We adopt the conservative approach of using a compensated density profile so as not to alter the background metric of the universe, which otherwise would be FLRW only asymptotically. The radius  $L$  of the inhomogeneity that is used in Eq. (2) is the radius at which the central over- or under-density has the transition to the surrounding compensating under- or over-dense shell. The radius  $L$  is hence smaller than the radius  $r_b$  which defines the radius of the total LTB patch, including both the central perturbation and its compensating shell.

In summary, the local structure is parametrized by the radius of the boundary  $r_b$  and the central curvature  $k_c$ . For any choice of these parameters, and for a  $\Lambda$ CDM cosmology (given by the background matter density, the curvature parameter  $k_b$ , the present-day background Hubble rate and a specific  $P(k)$ ), we can compute the probability of the existence of such a structure using Eq. (2) and compute the luminosity distance from an object to the central observer as a function of redshift.

**FLRW Observer's  $w(z)$**  Following [14] one can – given a luminosity distance-redshift relation in a homogeneous universe described by the FLRW metric – compute what the underlying  $w(z)$  of the dark-energy fluid is. There is an exact relation between  $w(z)$  and the first and second derivatives of the luminosity distance with respect to redshift and two more parameters,  $\Omega_k$  and  $\Omega_m$ . Therefore, the observer can derive  $w(z)$  if he/she knows the latter two parameters from other observations, and

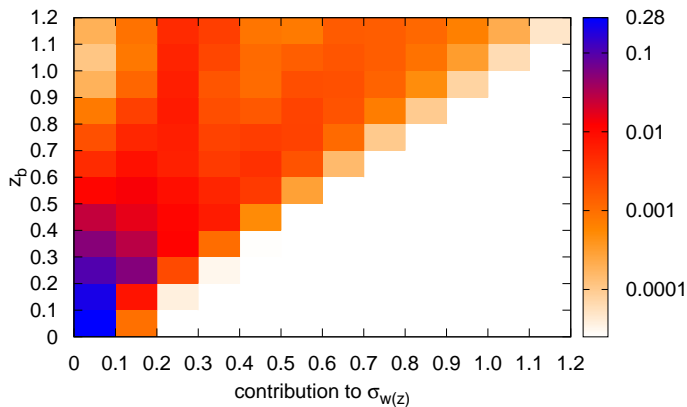


Figure 1. Matrix representation of the contribution of matter perturbations with a radius  $0.1(i-1) < z_b \leq 0.1i$  (the index  $i = 1..12$  labels rows) to the dispersion  $\sigma_{w(z)}$  at  $z_j = 0.05 + 0.1j$  (the index  $j = 1..12$  labels columns).

deduces the derivatives of the luminosity distance from SN observations. In the scenario studied here,  $w = -1$ . However, the inhomogeneities come into play and modify the luminosity distance-redshift relation. Consequently, an observer that erroneously assumes that the metric surrounding him/her is FLRW will in fact see a redshift-dependent  $w$  [8, 26–28].

In order to study this fundamental variation of the reconstructed  $w(z)$ , we first set the fiducial flat  $\Lambda$ CDM model to the WMAP7+LRG best-fit cosmology [29]. We then sample the  $\{r_b, k_c\}$  parameter space by building a Markov chain using Eq. (2) as likelihood, but restricted to a certain range in redshifts  $0.1(i-1) < z_b \leq 0.1i$  for  $i = 1..12$ , where  $z_b = z(r_b)$  is the apparent redshift at which an observer sees the radius  $r_b$  (see Eq. (7)). Next we compute from Eq. (8) the corresponding luminosity distances in the various realizations. We finally derive using Eq. (3) of [14] the (apparent)  $w(z)$  that an FLRW observer would infer. In this procedure we let the FLRW observer fix  $\Omega_m$  to the fiducial value as it cannot be determined from cosmological observations alone when allowing for an arbitrary equation of state of the dark energy [30]. The curvature could in principle be constrained by combining measurements of the distances with measurements of the expansion rate  $H(z)$ , but for simplicity we also fix  $\Omega_k$  to the fiducial (zero) value. The result of this analysis gives the variance in  $w(z)$  at the nodes  $z_j = 0.05 + 0.1j$  for  $j = 1..12$  induced by structures falling in the redshift bin  $0.1(i-1) < z_b \leq 0.1i$ , which is shown in Fig. 1. In Fig. 2 we show examples of typical  $w(z)$  evolutions as seen by the FLRW observer, affected each by one structure of an arbitrary size.

**Fitting the luminosity distance** However, usually one does not derive a fully general  $w(z)$  but fits a parameterized model to the distance data. Typical examples are a constant  $w$  or the linear model used for the DETF FoM mentioned in the introduction,  $w(a) = w_0 + (1-a)w_a$ .

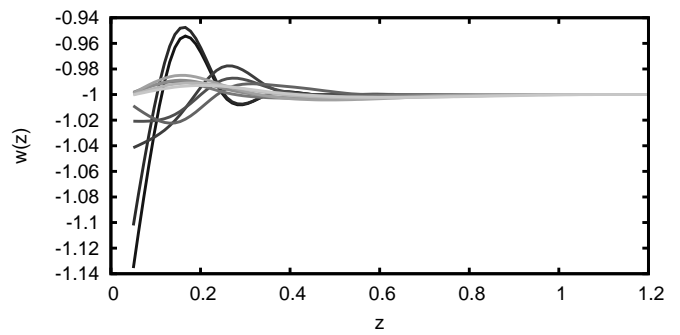


Figure 2. Examples of apparent evolution of the dark-energy equation of state that an FLRW observer would deduce from observations in a flat  $\Lambda$ CDM universe endowed with a local inhomogeneity coming from a post-inflationary gaussian density field.

As the impact of the local structure is strongest at low redshift, the variance of the fitted parameters will depend on the way the weight of distance data depends on redshift. For the purpose of our letter we choose a specific redshift distribution modeled to resemble the expected supernova distribution of the Dark Energy Survey (DES). Specifically, we weight the 12 redshift bins in the range  $0 < z \leq 1.2$  by using the binned rms scatter  $\sigma_{\text{bin}}$  of the Simulated DES Hybrid 10-field Survey reported in the third column of Table 14 of [31].

The basic approach is then as above: We fix again a fiducial cosmology with the same parameters as in the previous section (WMAP7+LRG) and sample again the parameter space describing the inhomogeneities in separate redshift bins  $0.1(i-1) < z_b \leq 0.1i$  using Eq. (2) as likelihood. We then fit the parameterized model to the data and determine the best-fit parameters  $\theta_i^*$  by minimizing the following  $\chi^2$ :

$$\chi^2(\theta_i) = \sum_{j=1..12} \frac{[m_{\text{hom}}(z_j; \theta_i) - m_{\text{inh}}(z_j)]^2}{\sigma_{\text{bin}}^2(z_j)}, \quad (11)$$

where  $z_j = 0.05 + 0.1j$ ,  $m_{\text{hom}}$  and  $m_{\text{inh}}$  are the distance moduli of the homogeneous  $\Lambda$ CDM model and of the inhomogeneous model, respectively (with the latter playing the role of the data in our context), and we marginalize analytically the likelihood  $\propto e^{-\chi^2/2}$  over an unknown offset. This time we can optionally also vary  $\Omega_k$  and  $\Omega_m$  as the parameterized model breaks the degeneracies.

In Fig. 3 we show the dispersion induced by a single structure of size  $z_b \leq 0.5$  on the fit parameters  $\{\Omega_m, w_0, w_a\}$  when the observer assumes that  $\Omega_k = 0$ . Since the effect does not change much for different structures up to  $z_b = 0.5$ , it is safe to consider the average variance induced by such structures. We compute the expected variance for such structures by combining the MCMC chains for the size bins up to  $z_b \leq 0.5$  by thinning them down to having the exact same number of points and then merging the resulting thinned chains.

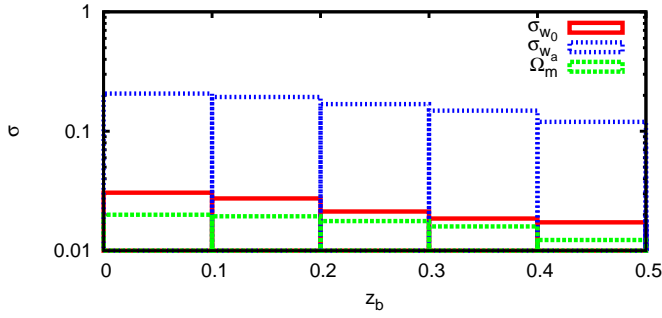


Figure 3. Dispersion induced by a single structure of size  $z_b \leq 0.5$  on the fit parameters  $\{\Omega_m, w_0, w_a\}$  when  $\Omega_k = 0$  is assumed. The last column in Table I gives the corresponding values when one averages the effect of these structures.

This leads to a single chain in which each size bin contributes with equal weight. We list in Table I the numerical values of the dispersions for the four different parameterized models. The variances listed in Table I show that the density perturbation around us, stemming from a standard inflationary spectrum and of unknown density, adds an extra uncertainty to these parameters. Unless a measurement of the local perturbation becomes possible in the future, we can never measure these parameters at a higher accuracy than the listed variances.

	$\{\Omega_m, \Omega_k\}$	$\{\Omega_m, w_0\}$	$\{\Omega_m, \Omega_k, w_0\}$	$\{\Omega_m, w_0, w_a\}$
$\sigma_{\Omega_m}$	0.013	0.0043	0.019	0.018
$\sigma_{\Omega_k}$	0.029	—	0.067	—
$\sigma_{w_0}$	—	0.020	0.057	0.025
$\sigma_{w_a}$	—	—	—	0.18

Table I. Intrinsic  $1\sigma$  uncertainties on fitted parameters for four different FLRW models. In each model all remaining cosmological parameters are fixed to the WMAP7+LRG best-fit cosmology. The free parameters are fitted to the inhomogeneous distance by minimizing the  $\chi^2$  of Eq. (11). The listed dispersions are the standard deviation of the posterior distribution on the given parameter, marginalized over the other free parameters.

Regarding the FLRW model where  $\{\Omega_m, w_0, w_a\}$  were left free, we observe that – based on these results – an experiment like DES can never determine  $w_0$  to a precision better than 2.5%, and  $w_a$  better than 18%. The Figure of Merit is defined as  $\text{FoM} = 1/A$  where  $A$  is the area bounded by the 95% c.l. contour on the  $\{w_0, w_a\}$  plane. The covariance matrix for  $\{w_0, w_a\}$  (which is the inverse of the Fisher matrix) for this intrinsic noise on the dark-energy equation of state is:

$$C_{\min} = \begin{pmatrix} \sigma_{w_0}^2 & \rho \sigma_{w_0} \sigma_{w_a} \\ \rho \sigma_{w_0} \sigma_{w_a} & \sigma_{w_a}^2 \end{pmatrix}, \quad (12)$$

where  $\sigma_{w_0}$  and  $\sigma_{w_a}$  are given in the last column of Table I and the correlation is  $\rho = -0.924$ . The area is

then  $A_{\min} = \pi \sqrt{\det C_{\min}} \Delta\chi^2 = \pi \sigma_{w_0} \sigma_{w_a} \sqrt{1 - \rho^2} \Delta\chi^2$  where  $\Delta\chi^2 \simeq 5.99$  for a 95% c.l. contour. We find  $1/\sqrt{\det C_{\min}} = 581$  and  $\text{FoM}_{\max} = 31$ .

**Conclusions** We have estimated in a conservative way the intrinsic uncertainty in the reconstruction of the dark-energy equation of state by means of distance measurements. Since we observe only one universe, this uncertainty will show up as a bias in distance measurements, and we propose that the scientific community use the results of this letter so as to include this extra source of error in their analysis. In particular, we give the covariance matrix  $C_{\min}$  for the linear model  $w(a) = w_0 + (1 - a)w_a$  used for the DETF FoM, which can be easily convolved with any other posterior distribution constraining the parameters  $w_0$  and  $w_a$ . Since the large-scale structure limits strongly the power of distance measurements as a probe of the nature of the dark energy and of the curvature of the universe, one may need to use data e.g. from galaxy surveys, weak lensing measurements or from the integrated Sachs-Wolfe effect to reduce its impact at least partially.

This analysis can be extended in a number of ways. For example, one may model the local inhomogeneity using non-spherically symmetric and non-compensated density profiles. Even more important would be the inclusion of non-local structures like superclusters and filaments which may increase the noise through lensing. According to results from second-order perturbation theory [28, 32] there is a scatter of about  $\sigma_\mu \approx 0.02$  or more in the luminosity distance even for  $z > 0.4$  which can lead to a percent-level variation for  $w$  also at high redshifts.

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